

5.0 TRANSECT METHODS

5.1 Introduction

One of the main handicaps faced by the practicing population ecologist is that there really is no wholly reliable census method for most wild animal populations. In spite of the intensive theoretical work done on capture-recapture methods, there yet remain various unresolved issues. Hence, as frequently remarked here, it is essential to use more than one method, and to do as much cross-checking and testing of assumptions as possible. It is, of course, easy to recommend such a course, but very difficult to follow it. For census methods, the only sure test of the underlying assumptions may in fact be to secure an absolute population count. Even then we are left with the question of sampling errors--an observed discrepancy may simply be due to chance alone.

Since the capture-recapture methods require at least one outright capture of a sample of animals, followed by one or more repeat observations (which may be visual only), they are necessarily expensive to use in practice. Catch-effort and change-in-ratio methods can only be used effectively on a harvested population. These limitations lead to a need for a method based solely on visual observation since it is often relatively inexpensive and rarely poses any threat to the population. For these reasons, and no doubt because of wider recognition of the difficulties with other methods, there has recently been a considerable interest in transect methods.

One of the brighter prospects for the future of transect methods is that it may be possible to avoid the pitfall posed by the "equal probability of selection" assumption required to apply elementary probability models. Unfortunately, some of the early work on transect methods included an equally untenable assumption, that individual animals are randomly and independently distributed over the study area. I prefer to adopt the working axiom that this is never the case, even when tests for departure from a Poisson distribution are "not significant." I will cheerfully abandon that viewpoint whenever the power of a test of randomness can be shown to be suitably large. Presently, a random distribution of individuals may have to be assumed for various features of secondary importance, such as obtaining an approximate notion (really a lower limit) of a variance for an estimate. However, bootstrapping offers promise for better variance estimates, with less difficulty.

Avoiding the assumption of a random spatial pattern of individuals requires that we substitute random location of transect lines. Systematically spaced lines are much easier to use and have other practical advantages. Not the least of these is the fact that randomly located lines may fall very close together so that running one such line can influence animals on a nearby line. Some ways to avoid this problem are discussed below. Although we will not try to go very deeply into the issue here, it should be remarked that the choice between random and systematic sampling for transect methods is not as simple a matter as for, say, plot sampling. For plot sampling, two features are of paramount importance. One is to avoid a systematic pattern that is correlated with a similar pattern in the material being sampled. The other is that variances obtained from systematic samples usually overestimate the true variance.

Some of the transect estimators depend on a probability model that holds strictly true if and only if the transect lines are indeed randomly located. This may place a considerable premium on adhering to random sampling. Possibly the effect of departing from that model may eventually turn out to be of minor importance. Until more work has been done of the "robustness" of the estimators, we will adhere to the requirement of random sampling, when this is at all feasible. If very large areas are to be covered, it may not be practicable to use anything but a systematic arrangement of transect lines. However, the most apparent problems with systematic samples apply to situations of a much smaller scale, not when lines are very widely spaced.

5.2 A classification of transect methods

Terminology for transect methods is not well-established. We will adhere to a usage that includes three main classes. The strip-transect is essentially a long narrow plot, on which it is basically assumed that all of the individuals present can be seen and tallied. As such, there is no important difference from plot sampling. Some modifications tend to make it more interesting and worth special attention. These include censusing marine mammals at sea, when individuals may submerge for varying periods of time, and thus escape enumeration. In many transect applications the mobility of individual animals is neglected. This is not feasible for those species that are observed when in motion, such as small birds. Thus another kind of modification needs to be considered.

In most census methods individual objects are regarded as points scattered around the map. Sometimes this abstraction either is not practicable or is inefficient. The investigator may be directly concerned with such quantities as the canopy coverage of shrubs or the volume of logs left lying in a cutover area. There is then an advantage in measuring the size of the object intercepted by the line; hence the descriptive term of line-intercept method.

The third class is perhaps best known, and includes the methods in which decreasing visibility of objects with distance away from the transect line has to be taken into account. We include all such methods under the general heading of line-transects. Some writers use the same term to apply to both strip-transects and line-intercepts. The terminology adopted here has the advantage of being reasonably explicit in descriptive terms.

Where mobile animals are concerned, one important distinction lies in whether or not the animal responds conspicuously to the observer's approach. One can then measure the flushing-distance, i.e., the straight-line distance between observer and animal at the time the animal "jumps" or "flushes," i.e., leaves cover. This is also designated in the literature as the radial distance or as the sighting distance. It is essential, however, to also measure the angle or the right-angle distance (i.e., the distance between the track line and the animal).

When detection depends mainly on the observer locating the animal or other object without the help of a flushing-response, there is reason to believe (cf. Robinette et al., 1974) that the flushing-distance models may not hold, and may lead to biased estimates. As alternate approach in such cases is based on

use of right-angle distances. It is thus useful to consider a dichotomous approach (Fig. 5.1) to the several classes of methods. It should help readers keep the various circumstances and conditions leading to the several classes of methods in mind. A recent development in which the observer remains at a point and estimates distances to surrounding objects has been known as the variable circular plot, and is largely treated by methods used for right-angle transects.

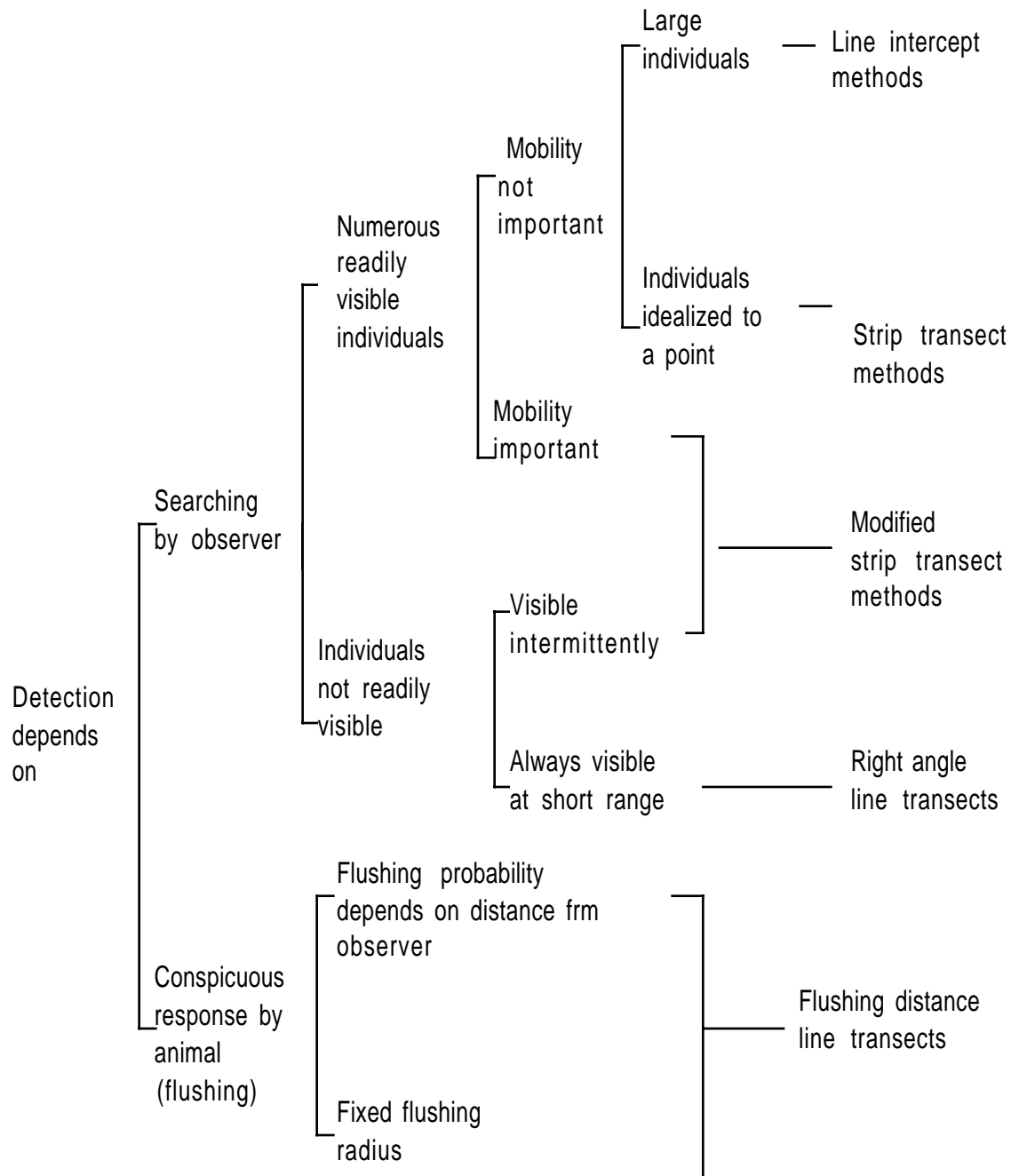


Fig. 5.1. A classification of transect methods.

5.3 The line-intercept method

The line-intercept technique has been used by plant ecologists for many years as a means of estimating "canopy-coverage." In that instance, the basis is simple and direct. All that is necessary is to measure the fraction of the total length of a given transect line that actually intercepts shrub canopies. The arrangement can be depicted as in Fig 5.2, which represents a rectangular study area having dimensions W and L , with a single transect (of length, L) intercepting two shrubs, for one of which the appropriate canopy-coverage measurement (l_i) is indicated. The technique can also be used for tree canopies by sighting upwards to find the margins of the canopy.

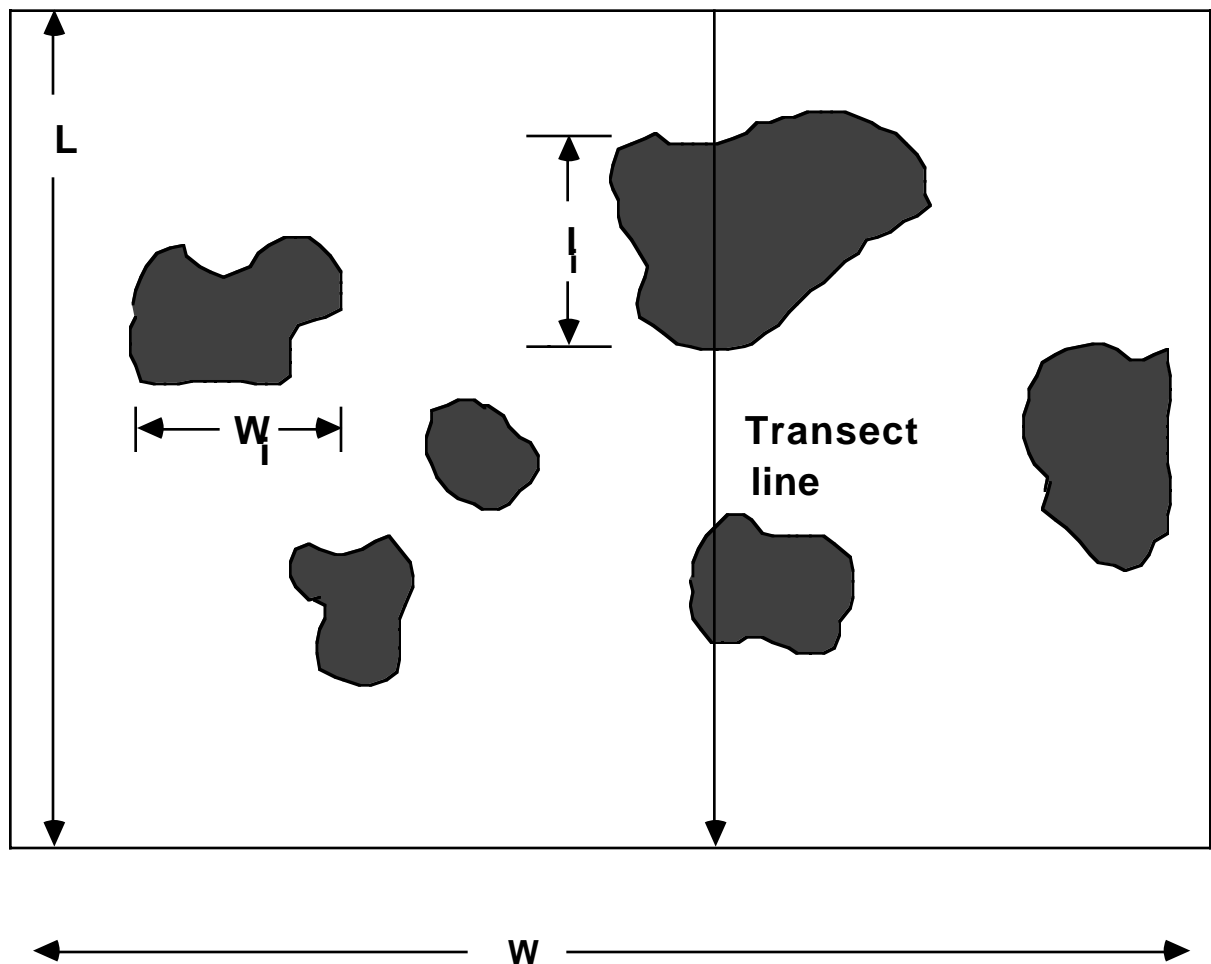


FIG. 5.2 Dimensions used in the line-intercept method. The shaded areas represent shrub canopies.

An unbiased estimate of canopy coverage is just the sum of the l_i observed on all of the transect lines divided by the total length of transects used. Unless there is some sort of regular pattern in the arrangement of the shrubs, very likely a systematic spacing of lines should not cause trouble in this situation. We repeat, however, that the basis of the results given in this chapter lies in randomized location of transect lines.

Two minor points may cause some trouble in actual practice. One is that the boundaries of the study area may intersect some shrubs. A useful rule, that should be decided on before the area is laid out, is to include such plants if they occur on, say, the northern and eastern boundary and exclude them on the other two boundaries. The second common problem is that many useful natural study areas (e.g., habitat types) are very irregular in shape. An easy way to deal with this kind of situation is to proceed as in Fig. 5.3. All that is needed is a baseline W that runs the full length of the area, and to utilize transects of variable length (L_i) with this length measured only within the study area. The calculations are illustrated in Example 5.1.

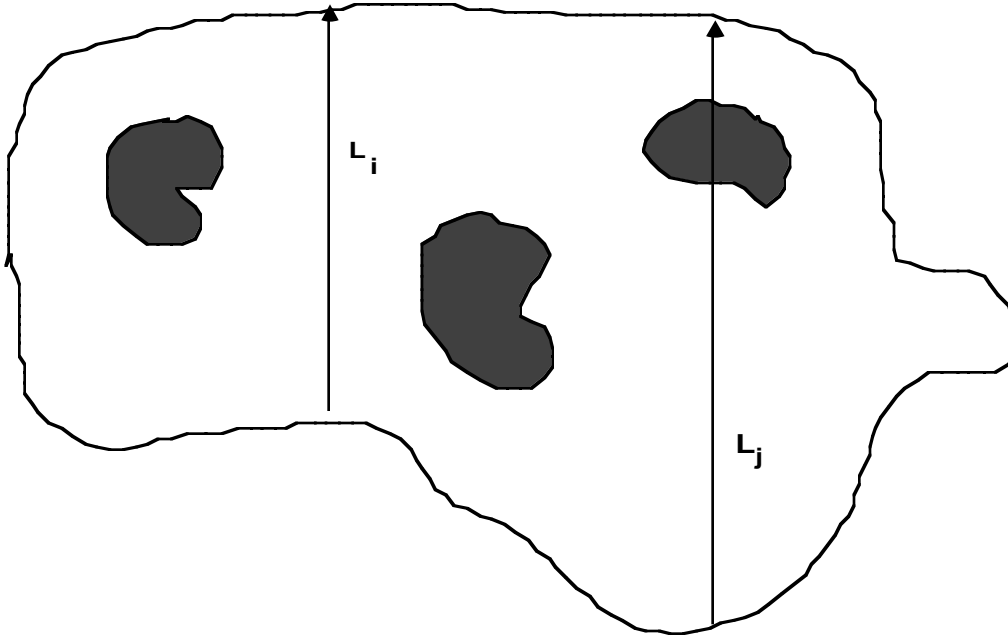


Fig. 5.3. Line-intercepts on an irregularly shaped area.

Estimates of the numbers or density (number per unit area) of individual plants have usually not been made by the plant ecologists in their use of the method. However, there is a simple way to obtain an unbiased estimate of density, although it "costs" an additional measurement. A biased estimate can be obtained without an extra measurement, and will be described first. McIntyre (1953) investigated the use of the measurement l_j for density estimation, and proposed several possible procedures. In using the length of the transect interception (l_j), he considered that the shrubs could be represented by a population of circles of varying diameter. Given random interceptions, it is then easy to derive a theoretical expression for length of intercepts which leads to the equation for density:

$$D = \frac{2}{\pi n L} \sum_{i=1}^m \frac{1}{l_i} \quad (5.1)$$

in which n stands for the number of transects of length L and m for the number of shrubs actually intercepted (for each of which l_j is measured). As already noted, the transects do not have to be of the same length. The only change is to replace nL by $\sum L_j$ in the denominator above.

Some trials with an artificial population led McIntyre (1953) to suggest that his method might well be useful for objects other than circles. Very likely the best results will be obtained for objects with smooth boundaries and few indentations or "scalloped" edges. This is because Eq. (5.1) uses reciprocals of the l_i , so that a few very short measurements will have a disproportionately large effect on the estimate. To avoid this problem, McIntyre recommended using the longest chord parallel to the transect line (and another equation). However, some better procedures are given below. Eq. (5.1) should mainly be used for an approximate notion of density when canopy coverage is the main purpose of the survey and an extra measurement is not justified.

An interesting alternative to McIntyre's approach can be described as "needle sampling" (DeVries 1974). It was originally developed for inventorying logs lying on the ground in cut-over areas. Instead of a circle, the object now is defined as a "needle" (which can be inscribed in a variety of only roughly elongate objects) and the famous results of "Buffon's needle problem" used to obtain a density estimate. The chief drawback is that the needles need to be oriented randomly, an assumption that may well be questioned in practice. More details appear in Example 5.2.

Example 5.1 Censusing prairie-dog dens

Line-intercepts were used to estimate the number of dens in a prairie-dog (Cynomys ludovicianus) colony by Eberhardt (1978b). The colony was elliptical in shape, with a long dimension of about 700 m and a maximum width of about 500 m. A systematic sample was used, with 9 transects spaced 66 m apart, and running across the narrower dimension of the area. The earth mounds at each den served in the same manner as shrub canopies in the usual application of the line-intercept method. For each mound intercepted by the transect line, measurements of the length of the interception (l_i) and the mound width (w_i), as shown in Figure 5.2. It should be noted that w_i is taken so as to measure the probability of interception for the mound, i.e., it is the distance between transects that just touch the right- and left-hand extremities of the mound.

The individual observations appear in Table 5.1, which also includes the distance between mound centers, or, at the ends of transects, the distance to the edge of the area grazed by the prairie dogs. This was regarded as the boundary of the study area. Calculations of density are thus for the grazed area immediately surrounding the mounds. Calculations on the basic data are summarized in Table 5.2. Proportion of the area covered by mounds is easily estimated, being just the total length of intercepts divided by the total length of transect lines. Thus for the first transect, it is:

$$p_i = 6.12/228.69 = 0.027.$$

For the entire area, the proportion covered is just the sum of all intercepts divided by the sum of transect lengths:

$$\hat{p} = \frac{\sum T_i}{\sum L_i} = 115.36/3578.9 = 0.0322.$$

This is a ratio estimate, for which a variance estimate is given in Chapter 4. Here T_i represents the total length of intercepts on the i th transect, i.e., $T_1 = 6.12$ m. The numerator could just as well be

written $\sum_i \sum_j l_{ij}$, where j denotes observations on a transect and i denotes the transect, but using the transect totals makes it easy to see that this expression has the same form as the ratio estimators of Chapter 4.

The finite population correction is neglected here since a small fraction of the population of mounds was actually tallied. Letting y = total interceptions (T_i) and x = transect length (L_i), the calculations are:

$$[CV(\hat{p})]^2 = \frac{1}{n} [c_{yy} + c_{xx} - 2c_{yx}] = \frac{1}{9} [0.1426 + 0.1507 - 2(0.1176)] = 0.00644$$

The estimated standard error for \hat{p} is then just $(0.00644)^{1/2}$ (0.0322) = 0.0032. The coefficients of variation are appreciably larger than recommended (in Chapter 4) for use of the ratio method. We thus propose that a simple approach might be used here, i.e., compute a variance directly from the proportions covered of the individual transects. This gives $p = 0.0329$ (averaging the transect values), with a standard error of 0.0028, so there is little difference from the ratio estimate.

Since widths (w_i) of the mounds were tallied, McIntyre's method for estimating density, Eq. (5.1), should not be used here. Apart from the factor of $2/\pi$ in the equation, calculations would proceed in exactly the same way as those utilizing widths, given next.

Although I recommend random sampling because most of the estimation procedures in transect work are based on random transect locations, the present example is one in which a systematic sample was taken. This was done mainly to study the pattern of spatial distribution of dens. With a systematic layout and distances between dens (Table 5.1), one can study the spatial arrangement of the dens. This is much harder to do with random transect locations, since random samples, especially relatively small ones, frequently leave sizeable gaps in spatial coverage.

In the present example, we can proceed in essentially the same manner for either random or systematic transect locations. The rationale differs somewhat, and needs to be mentioned for each case. It may be noted, too, that neither Eq. (5.2) or (5.3) is usable here since the transect lines are of variable length and the total area is not known. When the transects are randomly located, each individual transect yields an independent estimate of density, which can be calculated from Eq. (5.2), with $n = 1$. Using data from transect No. 1 (Table 5.1), we get (calculations in meters):

$$D = \frac{1}{L_i} \sum_{i=1}^{m_1} \frac{1}{w_i} \\ = \frac{1}{228.7} \left[\frac{1}{3.81} + \frac{1}{3.12} + \frac{1}{0.86} + \frac{1}{0.81} + \frac{1}{1.12} \right] = 0.0169 \text{ dens/m}^2$$

The same procedure can be used for each of the other transects, and the remaining question is one of how to combine 9 independent estimates (assuming, for illustration, that the transects had been randomly located along a baseline, as in Fig. 5.3). Averaging, and

computing a variance from the individual transect values, is both straightforward and legitimate.

The above procedure is often not very efficient, since it does not take into account the variation in transect lengths. A logical way to do this is just to weight the individual density estimates by the transect lengths, i.e., to calculate:

$$\hat{D}_{\text{overall}} = \frac{\sum_{i=1}^n L_i \hat{D}_i}{\sum_{i=1}^n L_i}$$

which is a ratio estimate just as in Chapter 4.

For a systematic sample, a somewhat different rationale might be used. This is because the uniform spacing of the transects permits viewing the area as being broken down into a number of strips of equal width. We then calculate an estimate of the number of dens in each strip, sum these, and divide by the total area (obtained by summing up the area of the individual strips). The procedure turns out to give exactly the same result as above, since a constant strip width is introduced in both numerator and denominator, and thus cancels out.

Using the data of Table 5.2 gives the following density estimate.

$$\hat{D}_{\text{overall}} = 70.013/3578.9 = 0.0196$$

The coefficient of variation is again estimated as in Chapter 4, without the finite population correction (for the reasons discussed above):

$$[CV(\hat{p})]^2 = \frac{1}{n} [c_{yy} + c_{xx} - 2c_{yx}] = 1/9[0.1698 + 0.1507 - 2(-.042)] = 0.0042$$

The standard error of the estimate is $(0.0449)^{1/2} (0.0196) = 0.0042$. These results are very similar to those for intercept length. The above example might be converted into a prairie-dog census method if the number of prairie-dogs inhabiting a representative sample of dens could be estimated.

Table 5.1. Spacing (d_i), intercept lengths (l_i) and mound widths (w_i) for 9 line-intercepts in a prairie-dog "town". Transect no. 1 was the westernmost transect. Spacing (d_i) in meters, other measurements in cm. The first d_i is distance to first mound from margin of the area and the last d_i is distance from last mound to the other margin of the study area.

# 1			# 2			# 3		
d_i	w_i	l_i	d_i	w_i	l_i	d_i	w_i	l_i
2.84	381	127	16.06	130	74	138.0	124	107
46.3	312	114	6.62	290	236	39.69	117	104
39.69	86	152	27.4	132	96	67.1	218	124
5.67	81	84	6.62	109	74	9.45	168	58
20.79	112	<u>135</u>	35.91	401	224	1.89	160	74
<u>113.4</u>		612	124.74	282	160	34.02	368	157
228.69			41.58	274	<u>213</u>	27.4	198	<u>38</u>
			<u>28.35</u>		1077	<u>9.45</u>		662
			287.28			326.97		

# 4			# 5			# 6		
di	wi	li	di	wi	li	di	wi	li
189.00	112	74.00	68.98	76	58	5.67	94	81
4.72	274	170.00	24.57	350	244	11.34	175	170
66.15	89	66.00	37.8	127	74	2.84	71	84
29.30	256	185.00	34.02	196	76	27.40	102	79
22.68	79	41.00	23.62	140	107	65.20	183	74
10.40	376	208.00	17.01	61	58	60.48	158	132
15.12	406	109.00	5.67	117	112	20.79	74	48
9.45	340	203.00	31.18	86	89	12.28	117	112
34.02	249	310.00	20.79	175	168	17.96	91	48
69.93	96	81.00	7.56	229	203	26.46	216	119
8.50	389	338.00	20.79	163	163	9.45	175	122
25.52	46	46.00	12.28	274	193	10.40	114	81
5.67	89	<u>81.00</u>	72.76	201	160	128.52	163	53
<u>75.60</u>		<u>1912.0</u>	15.12	198	<u>109</u>	3.78	56	<u>112</u>
566.06			<u>22.68</u>		1814	<u>20.79</u>		1315
			414.83			423.36		

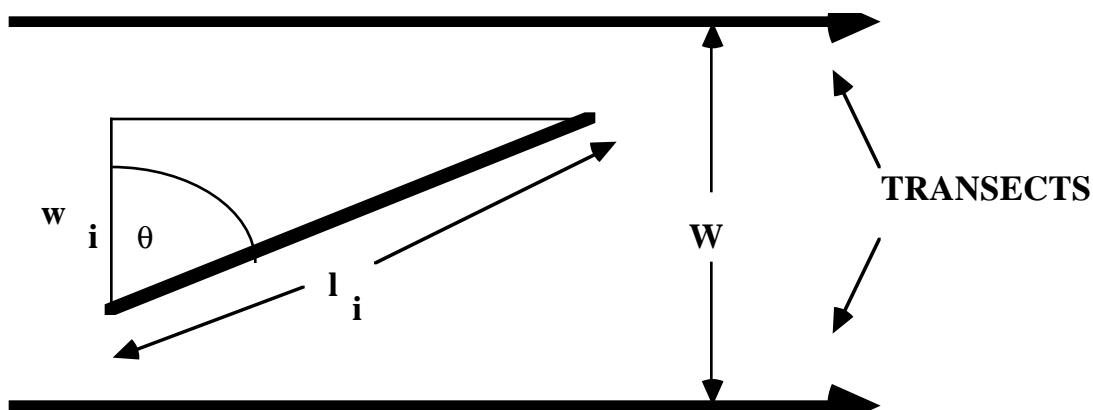
# 7			# 8			# 9		
di	wi	li	di	wi	li	di	wi	li
12.28	292	158	28.35	109	99	19.84	183	137
10.40	132	79	83.16	163	163	26.46	102	104
7.56	117	41	77.49	84	51	7.56	84	137
49.14	142	135	10.40	142	99	15.12	300	114
33.08	239	198	11.34	330	91	5.67	277	127
37.80	91	91	89.78	249	135	46.3	112	84
6.62	127	183	43.47	325	183	39.69	56	51
32.13	91	46	7.56	198	122	40.64	66	84
51.98	221	142	2.84	74	79	11.34	86	147
17.96	196	140	11.34	147	<u>132</u>	1.89	244	<u>185</u>
29.30	117	66	<u>9.45</u>		1154	<u>37.8</u>		1170
108.68	292	163	375.18			252.31		
274.00	287	274						
14.18	140	<u>104</u>						
<u>18.90</u>		1820						
704.01								

Table 5.2. Summary of line intercept data for a prairie-dog "town"

Transect number	Length of of transect L_i by mounds	Proportion of area covered intercepted	Number of mounds	Sum of intercepts T_i	Density estimates D_i
1	228.7	0.027	5	6.12	0.0169
2	287.3	0.037	7	10.77	0.0131
3	327.0	0.020	7	6.62	0.0126
4	566.1	0.034	13	19.12	0.0174
5	414.8	0.044	14	18.14	0.0247
6	423.4	0.031	14	13.15	0.0303
7	704.1	0.026	14	18.20	0.0132
8	375.2	0.031	10	11.54	0.0186
9	252.3	0.046	10	11.70	0.0364
	3578.9		94	115.36	

Example 5.2 "Needle" sampling

As with most similar sampling problems, this one is most readily conceptualized in reverse of what happens in practice. That is, we lay out the sampling scheme and then introduce, at random, the objects to be sampled. Here we suppose a systematic sampling pattern of parallel transect is laid out, and long, narrow objects of length l_i ("needles") are randomly distributed over the area. Let the spacing between the objects be W , and assume for simplicity, that $l_i \leq W$, i.e., that none of the "needles" is longer than the interval between transects. The relevant measurements appear in the figure below. A "needle" of length l_i is thrown randomly onto a field of parallel transect. The probability that it intercepts a transect depends on w_i , which in turn depends on the angle (θ) that the needle happens to assume.



Dimensions used in "needle" sampling.

We can write the probability of interception for a needle of given length (l_i) as:

$$P = \Pr\{\text{interception}\} = \frac{w_i}{W} = \frac{l_i \cos \theta}{W}$$

Since the angle (θ) is assumed to be randomly determined, it has a uniform distribution between 0 and 90° , or between 0 and $\pi/2$ in radians. Hence the frequency distribution of θ is:

$$f(\theta) = \frac{2}{\pi} d\theta$$

The expected value of p is then:

$$E(P) = \frac{2l_i}{\pi W} \int_0^{\pi/2} \cos \theta d\theta = \frac{2l_i}{\pi W}$$

If n needles are observed to intercept the transects, a simple estimate of the total number (N) of needles in the sampled population is:

$$\hat{N} = n/\bar{p} = \frac{n\pi W}{2} \sum_{i=1}^n \frac{1}{l_i}$$

and the estimated density of needles is:

$$D = \frac{n\pi}{2L} \sum_{i=1}^n \frac{1}{l_i}$$

The main problem with the method is that it is seldom safe to assume that the needles are randomly distributed. We thus recommend measuring w_i directly, and utilizing the equations given in the text for density estimation based on w_i . Students who want a demonstration of the method can readily construct one with a handful of kitchen matches scattered on a hardwood or tiled floor.

5.4 Length-biased sampling

The main issue in estimating shrub density from canopy measurement is one that is common to a very much wider class of sampling problems. Cox (1962, 1969) has used the highly descriptive term "length-biased sampling" to characterize procedures in which the probability of sampling a particular element in the population is proportional to some dimension of that element. Such a sample is by no means representative of the population, being very much biased towards individuals having the greater "lengths." In the present case, it is readily evident (Fig. 5.2) that the probability that a given shrub will be included in a sample taken by the intercept method depends on how "wide" it is with respect to the baseline (W) of the study area. The relevant measurement on the shrub is thus w_i (Fig. 5.2). It should be noted that w_i is the distance between tangent lines drawn parallel to the transect at the right- and left-hand extremities of the canopy.

The probability that a given shrub will be intercepted by the transect line is just w_i/W , on either Fig. 5.2 or 5.3. By measuring w_i accurately, one can thus determine the exact probability that a given shrub intercepted by the transect would be observed, before the transect line was selected. Given the probability of interception for each element observed in the sample, a straightforward argument can be constructed to derive a density estimate (Eberhardt 1978b). The principal equation is:

$$D = \frac{1}{nL} \sum_{i=1}^m \frac{1}{w_i} \quad (5.2)$$

As in Eq. 5.1, m is the number of objects intercepted (and measured) on n transect lines randomly placed in a rectangular area of dimensions W and L .

When the study area is not rectangular, a baseline W can be constructed as indicated in connection with Fig. 5.3, and density estimated from:

$$D = \frac{W}{nA} \sum_{i=1}^m \frac{1}{w_i} \quad (5.3)$$

where A represents the area of the study plot expressed in the same units (e.g., square meters) as the linear measurements (w_i and W). A useful approach when the area is not known is given in Example 5.1, which also illustrates variance calculations. Estimates of N , the total population are, of course, readily obtained from Eqs. 5.1 to 5.3 by multiplying by the area.

Lucas and Seber (1977) have derived equations comparable to those above, but use a different transect layout. They require that the transects be of short length, and both randomly located and randomly oriented with respect to the baseline. They obtain theoretical variance formulas for some circumstances. However, in the present state of theoretical and practical knowledge, it seems advisable to use variances estimated from replicated or interpenetrating sampling, as in Section 5.12 (below), or by the ratio method of Example 5.1.

The above method can be extended to deal with objects other than shrub canopies, and to aggregations of animals or patches of vegetation, so long as the identity and boundaries of each such "object" can be uniquely defined. It is also possible to substantially enlarge the area for interception of a given object. A method for doing this is well-known to foresters as Bitterlich's method. An "angle-gauge" is used to determine whether or not the apparent diameter of a tree is greater than a fixed angle, and thus whether or not the tree should be included in a sample. Readers not familiar with the method can simulate the field operation by extending an arm with the thumb in an upright position. If portions of an object (tree, rock, sign, etc.) protrude on both sides of one's thumb, then that object is "in" the sample. If the observer now moves away from the object until its margins just barely protrude beyond the sides of the "gauge" (thumb), then that position delineates the boundary of the interception area (Fig. 5.4). Circular objects like trees will have a circular boundary, but irregular objects will have an asymmetric boundary.

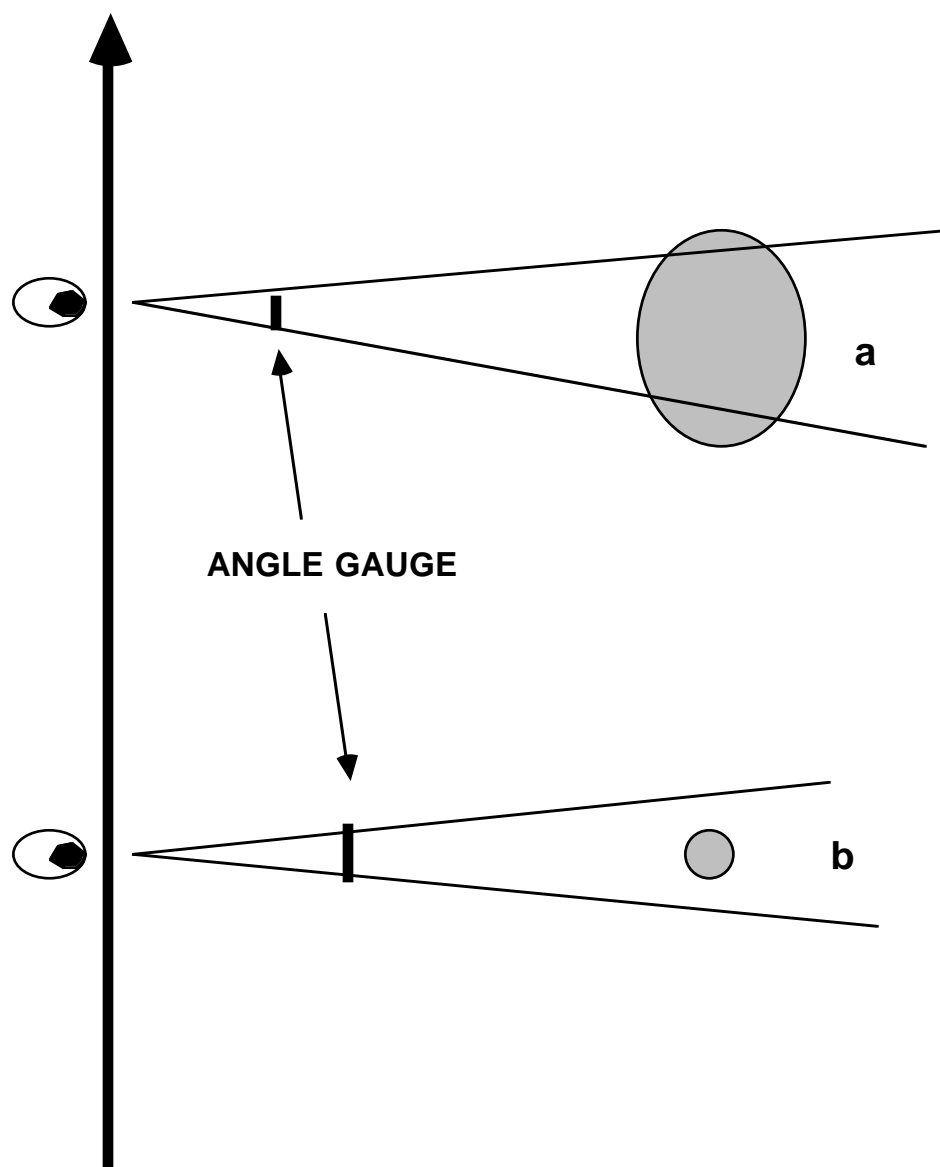


Fig. 5.4. Use of an "angle-gauge". Objects are "in" the sample when, as in (a), the sides protrude beyond the gauge. When the gauge blocks the object from view, as in (b), then it is not included in the sample.

Although Bitterlich's method is normally used only at fixed sampling points, it can be utilized as a transect method, as was proposed by Strand (1958). However, this will usually only be practicable for relatively rare objects, since "intersection" has to be determined by use of an angle-gauge as each object comes into a right-angle position on the transect line. The method might thus be most useful for something like a survey of den-trees in wildlife management.

Density may not be the main objective in some studies. When the volume, weight, or some other measurement is to be estimated, a simple ratio method can be used, and illustrated in Example 5.3.

Example 5.3 Auxiliary measurements

Often the primary objective of a study will be to estimate some attribute other than density, or in addition to density. Thus foresters are usually also concerned with basal area and volume of timber, while ecologists often want to estimate the biomass (total Weight) of vegetation. Methods for securing such estimates by ratio estimation were given in Chapter 4. A related method based on line-intercepts can readily be derived. Let X_i be the "auxiliary" measurement, such as weight or volume of the i^{th} object intercepted. A well-known way to estimate the average value of a sample of such objects is simply to "weight" each object inversely as the probability that it is included in the sample. Since this probability is proportional to w_i , we get the

$$\bar{X}_j = \frac{1}{m} \sum_{i=1}^m \frac{x_i}{w_i}$$

simple result:

If the above estimate is regarded as the estimated average on the j^{th} transect, then variable transect lengths can be adjusted for just as was done in Example 5.1, i.e.,:

$$\bar{x}_{\text{overall}} = \sum_{j=1}^n L_j \bar{x}_j / \sum_{j=1}^n L_j$$

and the same approach can be taken to obtaining a variance estimate (ratio method).

5.5 Flushing-distance line transects

In the line-transect method, the objects being censused are considered to be dimensionless points, and the probability of detection is assumed to be measured by use of distances between observer and object. Some trigonometry is involved, based on the distances and angle illustrated in Fig. 5.5. Just which measurements are taken will depend considerably on the particular field situation. The essential measurements for most purposes are r , the sighting-distance (also called radial distance or flushing-distance), and x , the right-angle distance. From simple trigonometry, any pair of the possible measurements can be used to calculate the others. However, precautions need to be taken to avoid measurement errors. I strongly recommend against visual estimation of either distances or the included angle (θ).

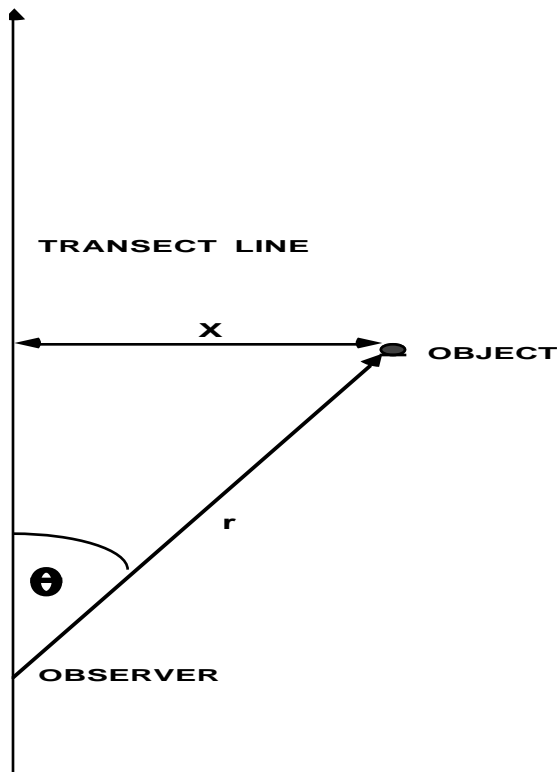


Fig. 5.5. Relevant measurements for the line-transect method. The vertical arrow shows the observer's path along a transect line.

In the flushing-distance model, the distance (r) between observer and animal at the time the animal flushes is the essential measurement. Since a test of the validity of the model is based on x/r (which is the sine of θ), these distances need to be measured as accurately as possible. When the right-angle distance method (described below) is used, only the distance x is utilized. A model for evaluating relative errors in measurements is described in Example 5.4.

Two basic flushing-distance line transect models have been proposed (Eberhardt 1968b). In one model it is assumed that the flushing-distance is fixed, i.e., that the individual animal flushes as soon as the observer crosses the boundary of a circle with radius equal to this fixed distance. This model is due to Hayne (1949), who noted that the fixed distance does not need to be assumed to be a permanent characteristic of the individual animal. The necessary assumption is that each animal on a census area has a fixed flushing-distance during the time when a given randomly located transect is run. In many circumstances it seems quite likely that the flushing-distance will depend very much on characteristics of the particular location in which an animal is resting.

The fixed-distance model permits a simple and direct analysis, proceeding in the same manner as for the line intercept method. The shrub canopy is now replaced by a circle of radius r , and it is assumed that the flushing distance (r) is measured accurately for each animal seen. It is also assumed that animals flush independently, i.e., that startling one animal does not change the behavior of the others. Analysis of the fixed flushing-distance

model then requires only noting that $w_i = 2r_i$, that is, w_i is the diameter of a circle of radius r_i . In analogy with Eq. (5.2), we now have:

$$D = \frac{1}{2nL} \sum_{i=1}^m \frac{1}{r_i} \quad (5.4)$$

with n the number of transects and m the number of observations as before. Irregular-shaped census areas can also be dealt with in the same manner as with line intercepts.

In some cases, groups of animals may flush together, as with broods of grouse, or flocks of small birds. If it can be shown, from field data, that flushing radius and group size are independent, it may be possible to use Eq. (5.4) to estimate the density of groups and multiply that estimate by average group size. If group size and flushing radius are correlated, one can still estimate the number of groups, but the average of group sizes is a biased estimator of the population mean.

As Hayne (1949) indicated, the expected flushing angle is 32.7° . A variety of field studies have yielded average angles that are close to this value for animals that "flush." Robinette et al. (1974), working mostly with animals that do not flush and inanimate objects, obtained wider mean angles. The underlying theory (cf. Eberhardt 1978b) shows that the frequency distribution of the ratio (x/r) of right-angle distances (x) to flushing-distances (r) should be that of the uniform distribution. Hence a simple chi-square test (Example 5.5) can be used to check on the validity of the model. If the test shows significant deviations from the hypothesis of a uniform distribution of x/r , then the best advice presently available is to utilize right-angle distances, as described below.

In the second model it is assumed that the instantaneous probability of flushing is a function of the current distance between observer and animal. It seems quite reasonable to assume flushing probability to increase steadily as the observer approaches, being nearly zero at a long distance and approaching unity in the immediate neighborhood of the animal. One might expect that an animal registers a variety of auditory and visual cues from an observer's approach, and that the cumulative effect of those cues results in an increased probability of flushing. Such a model is conveniently labelled the variable-distance model.

It does not seem likely that the two models can be distinguished on the basis of field observations. Either will lead to a frequency distribution of flushing distances, being based on a population distribution of flushing radii in the fixed-distance model, and on realizations of the probability model in the variable-distance case. Details of the theory appear in Eberhardt (1978a), and lead to the conclusion that Eq. (5.4) should be used for animals that flush. The theory also shows that flushing-distance (r) and flushing-angle (θ) should be independently distributed. Hence a useful further check of conformity to the flushing-distance model is to plot r and θ to see if there is any suggestion of association. Spearman's rank correlation coefficient might be used to test for correlation between r and θ (see, for example, Snedecor and Cochran 1967).

The fixed-distance model can be used to show that there appears to be a loss of efficiency (i.e., a larger variance results) if right-angle distances are used when the flushing-distance model holds. It should be noted that the current authoritative reference on "distance sampling" (Buckland et al. 1993) has dropped the idea of using sighting distances. They remarked that "Hayne's (1949) method is poor if $\bar{\theta}$ is not approximately 32.7° and may not perform well even if $\bar{\theta}$ falls close to this value, i.e., is not a robust method." Consequently they use only right-angle distances.

Example 5.4 Errors of measurement in line transects

Wherever possible, the relevant measurement for line transect estimation should be measured directly and as accurately as possible. However, it may at times be necessary to calculate the appropriate measurement by trigonometry on the pairs of the measurements of Fig. 5.5. Anyone doing this should be aware that the effect of incremental errors may vary considerably, depending on the particular pairs used. Suppose θ and x (Fig. 5.5) are measured and r is calculated as $r = x \sin \theta$. Then we note that $dr = x \cos \theta d\theta$, so that an incremental error ($d\theta$) in measuring θ results in a corresponding incremental error (dr) in the estimate of r . The absolute relative error in r is:

$$\left| \frac{dr}{r} \right| = \frac{\cos \theta}{\sin \theta} d\theta$$

when $\theta = 50^\circ$, $\frac{\cos \theta}{\sin \theta} = 11.43$, while for $\theta = 45^\circ$, $\frac{\cos \theta}{\sin \theta} = 1$, and for $\theta = 60^\circ$,

$\frac{\cos \theta}{\sin \theta} = 0.58$. Consequently, errors at small angles can have rather serious effects.

The above approach can be used to evaluate other arrangements, and a logical extension would be to explore the effects of errors on the final estimate by incorporating the theoretical frequency distribution. Doing so in detail calls for a knowledge of likely incremental errors ($d\theta$) at various angles, but this has not been investigated yet, to my knowledge. However, since the theoretical frequency distribution of angles is proportional to $\cos \theta$ (Eberhardt 1978b), it is obvious that errors at small angles ought to be avoided.

Too often, field data show evidence of gross errors. These appear in histogram plots of angles and distances as a tendency for measurements to pile up at angle like 0° , 30° , 45° and 90° , and for distances to be similarly grouped. "Trial runs" or pilot surveys are useful devices for catching such tendencies and training observers.

5.6 Right-angle distance line transects

When detection depends on the observer, it is unlikely that the flushing-distance (now sighting-distance) models can be expected to hold. The major summary of field experience is that of Robinette et al. (1974) and suggests that these models do not hold for animals that do not flush and for some inanimate objects. One prospect that needs study is that the manner in which observers scan ahead as they move along the transect may well influence the data. For the present, the safest course in circumstances where detection depends on the observer is to resort to use of right-angle distances,

and, as noted above, only the use of right-angle distances is recommended in the recent literature (Buckland et al. 1993). This may well require larger samples because of an added component of variability in using right-angle distances. Hence the need for research to determine whether sighting-distance models might be used, if suitable precautions are taken (in particular, advance surveys should show that the mean sighting angle is very close to 32.7 degrees). As noted above, only the use of right-angle distances is recommended in the recent literature (Buckland et al. 1993).

Supposing that the conservative course is chosen, i.e., that the right-angle distances are to be used, there then is the question of how to estimate density from such data.

A convenient frame of reference is that of Eberhardt (1968b). We again suppose that the study area is rectangular in shape as in Fig. 5.6 with a baseline of length W . It is assumed that virtually all of the observations made from a given transect line (represented by the solid line in Fig. 5.6) fall within a distance Z on either side of the transect line, and thus within the shaded area of Fig. 5.6. Hence if Z is small relative to W we can neglect most boundary problems. As suggested before, one can adopt the convention that observations made outside the study area on two boundaries will be included, and those outside of the other two boundaries will be neglected. So long as Z is quite small relative to W this approach should serve to deal with irregular shaped areas. To simplify the presentation, we now "fold" the left-hand side of the shaded area over onto the right-hand side and depict the actual observations of positions of observed individuals as in Fig. 5.7. If we then project these positions down onto a baseline, as shown by lines in Fig 5.7, we can analyze the data in terms of right-angle distances alone. The mathematical results then used (Eberhardt 1968b) are those of Parzen (1972). However, instead of an "intensity function," we use a "visibility curve," $g(x)$, as in Fig. 5.8. The essential features are that the probability of sighting an animal directly on the transect line shall be unity ($g(0) = 1.0$), and that the curve decrease smoothly away from the transect line. Further theoretical details appear in Burnham and Anderson (1976), Eberhardt (1968, 1978b) and in Buckland et al. (1993).

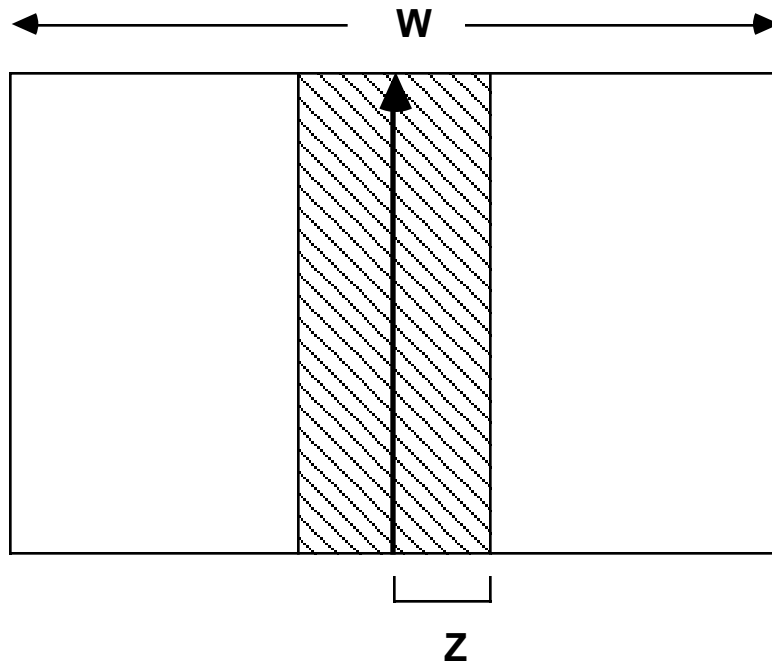


Fig. 5.6. Restricted area (shaded) used in many right-angle line transect methods.

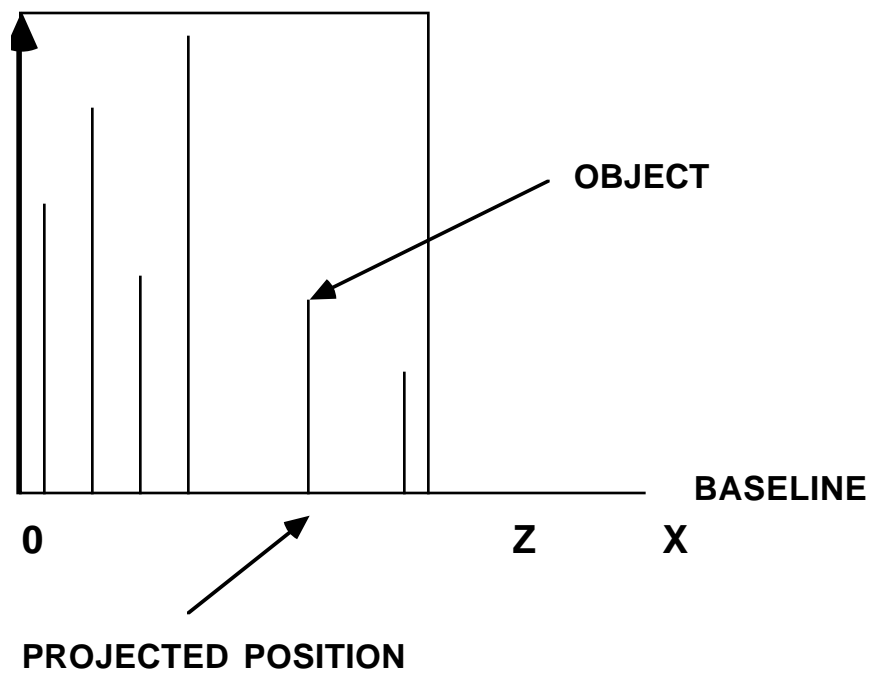


Fig. 5.7. Projection of observed positions on to a baseline.

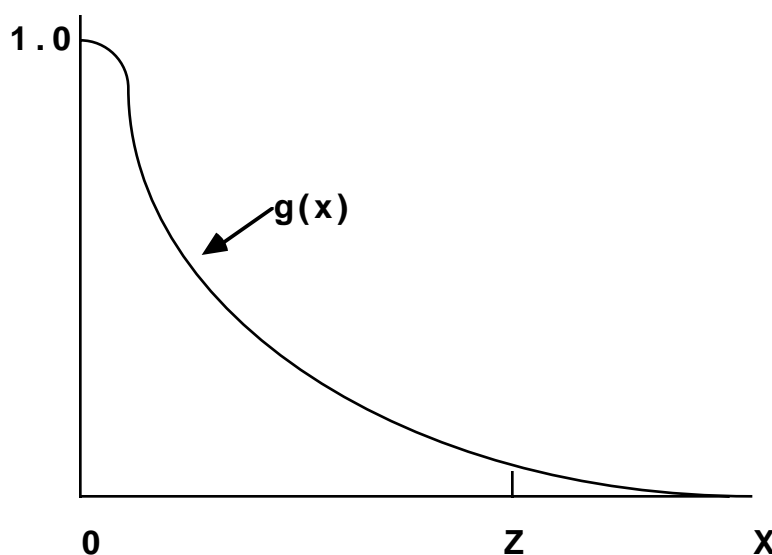


Fig. 5.8. A "visibility curve", $g(x)$, showing truncation imposed by neglecting observations beyond z .

The visibility curve of Fig. 5.8 is then the underlying model that generates the actual right-angle distances associated with a particular transect, represented by projecting to the baseline of Fig. 5.7. The position of Z in Figures 5.6 to 5.8 is arbitrary. It needs to be such that most, but not all, of the actual observations fall to the left of Z , when the entire set of data from a given study are considered. The actual selection of Z will be discussed below.

The visibility curve is not itself a frequency distribution, but it can be converted to such a distribution if it is divided by a constant that is the integral of $g(x)$, as shown by Burnham and Anderson (1976) and Eberhardt (1978b). Thus we have:

$$f(x) = \frac{g(x)}{\mu} \quad (0 \leq x \leq \infty) \quad (5.5)$$

where

$$\mu = \int_{x=0}^{\infty} g(x) dx$$

A simple example of a visibility curve is the negative exponential as used by Gates et al. (1968) and Gates (1969). They found that it fitted data on flushing of ruffed grouse (Bonasa umbellus) quite satisfactorily, and gave estimating equations for both flushing-distance and right-angle distance data. An objection to this curve, however, is that it drops off at a constant rate. As suggested by Eberhardt (1968b), a more logical curve would be one that is nearly flat near the transect line, dropping off sharply some distance from the line, and then "tailing off" more gradually. Such a curve accommodates both the realistic assumption that a narrow strip census is feasible (i.e., that nearly all animals will be seen on a narrow strip centered on the transect line) and the observational fact that a few animals are seen at some considerable distances from the transect line. One curve fitting this requirement is the "reversed logistic" proposed by Eberhardt (1968) and described in more detail by Eberhardt (1978b).

The negative exponential curve has one parameter while the reversed logistic has two. If one parameter of the latter curve is very small, it becomes virtually indistinguishable from the negative exponential. This property was used by Eberhardt (1978b) to explore the effect of small deviation from the negative exponential on the resulting density estimates. The simulations conducted by Eberhardt (1978b) yielded biases (overestimates) of 17 percent and 50 percent if the true model were one of the two reversed logistics but the negative exponential were assumed to be the appropriate model. Consequently we do not recommend assumption of the negative exponential model.

A variety of other models have been proposed in the literature. The half-normal (Hemingway 1971) is a one-parameter model having the shape suggested above as appropriate. Anderson et al. (1978) have proposed a log-linear model, while Pollack (1978) presents an exponential power series model. Both of these "families" of models include the negative exponential and half-normal and provide considerable flexibility. The immediate problem is a lack of published experience covering a variety of field data. We will thus not try to make any specific recommendations about the use of particular frequency distributions.

One of the several recent developments in frequency distribution models is the "Fourier Series" estimator of Crain et al. (1978). It provides a highly flexible model that may be expected to give very good fits to field data. Both theoretical and simulation studies were employed by Crain et al. (1978) to show that the method has relatively small bias and high efficiency. That work has been followed up in detail, with several new models, and the results published in "Distance Sampling" by S. T. Buckland, D. R. Anderson, K. P. Burnham, and J. L. Laake (1993). Computations are available in the program DISTANCE which is available on the worldwide web along with a comprehensive manual and the full text of the book by Buckland et al.

Example 5.5 Testing flushing-angles

A simple test is available to check whether observed angles are in conformity with the underlying theory. The test is actually based on the distribution of $\sin\theta$, and holds for either the fixed or variable flushing-distance model (Eberhardt 1978b). It is, however, most readily derived for the fixed flushing distance model. From Fig. 5.5, $\sin \theta = x/r$. Consider a fixed flushing radius of r . Given that the animal is flushed (i.e., that the transect passes through a circle of radius r about the animal), and that transects are randomly located, it is evident that x will take on any distance between 0 and r with equal probability. Hence the distribution of x/r is uniform over the interval 0 to 1. A simple test is then a chi-square test. Divide the interval from 0 to 1 into equal sub-intervals, with the number selected so that the smallest expected number is about 5, and tally the observations of x/r by intervals. An example (from Eberhardt 1978b), appears in Table 5.3.

There are 84 observations, and 10 subintervals were used, so that the expected number in each interval is 8.4. The chi-square test is then:

$$\text{Chi-square} = \sum_{i=1}^{10} \frac{[\text{observed number} - 8.4]^2}{8.4}$$

It is worthwhile to tabulate individual deviations and chi-square calculations (as in Table 5.3), so that any aberrant observations can be identified if the test shows statistical significance. In the present example, the chi-square value (10.73) is well below the 95 percent significance level (18.31) for 10 degrees of freedom. Students should note that 10 degrees of freedom are used here, because the expected value is obtained independently from the data.

Table 5.3 Chi-square test for uniformity of $\sin \theta$ data for a census of the side-blotched lizard.

<u>Interval</u> <u>($\sin \theta = x/r$)</u>	<u>Number of</u> <u>observations</u>	<u>Deviations from</u> <u>expected number</u>	<u>Chi-square</u> <u>value</u>
0.00-0.10	10	+1.6	0.30
0.10-0.20	7	-1.4	0.23
0.20-0.30	8	-0.4	0.02
0.30-0.40	15	+6.6	5.18
0.40-0.50	10	+1.6	0.30
0.50-0.60	10	+1.6	0.30
0.60-0.70	6	-2.4	0.68
0.70-0.80	4	-4.4	2.30
0.80-0.90	9	+0.6	0.04
0.90-1.00	5	-3.4	1.38
	<hr/> 84	<hr/> 0.0	<hr/> 10.73

5.7 Density Estimation

The generally accepted estimator for right-angle line transect models is (Seber 1982), Buckland et al. (1993):

$$\hat{D} = \frac{m}{2L} \left(\frac{1}{\hat{\mu}} \right) \quad (5.6)$$

where m is the number of objects observed, and L is the (total) length of transect on which the m objects are observed. The estimate of the reciprocal of μ is calculated from the observed distances. This is done by noting that, in Eq. (5.5), $f(0) = 1/\mu$. Thus the main objective of the various methods is to obtain an estimate of the frequency of observations "on" the transect line, or $f(0)$. Consequently, an equivalent form of Eq. (5.6) is just:

$$\hat{D} = \frac{m}{2L} \hat{f}(0) \quad (5.7)$$

Looking back to Eq. (5.4), it may be observed the Hayne's (1949) estimator is of this form, except that $f(0)$ or the reciprocal of μ is estimated from the average reciprocal of flushing distances, i.e.:

$$D = \frac{m}{2nL} \left[\frac{1}{m} \sum_{i=1}^m \frac{1}{r_i} \right] = \frac{1}{2nL} \sum_{i=1}^m \frac{1}{r_i}$$

and that n transects, each of fixed length L , were used.

Since, as we have already remarked, the "state of the art" and the theory of line transect are now described in detail by Buckland et al. (1993), we will not attempt to review all of the currently used methods. Some examples appear in Example 5.6 and I recommend consulting the current literature for recent improvements. I suggest use of the "distribution-free" methods of the next section as a check on any other method used.

Example 5.6 Density estimation for line transects

Calculations for two of the methods will be illustrated on the set of data in Table 5.4. These data come from actual observations made in a line transect study (Eberhardt 1978b) of the side-blotched lizard (Uta stansburiana). An artificial grouping of the data into 8 transects has been used here as a device to illustrate variance calculations. Since these data appear to conform to the theoretical model for animals that flush, it may be possible to use Hayne's method, Eq. (5.4). It may be remarked here that the "flush" exhibited by these animals is a dart for cover, and that nearly all sightings result from this cue, as basking animals are not readily seen before they move.

Using Eq. (5.4) gives the results of the summary table (Table 5.5). The equation is used with $n = 1$ for individual transects, i.e.,:

$$\hat{D} = \frac{1}{2L_i} \sum_{i=1}^m \frac{1}{r_i}$$

The individual transect results can be combined with the ratio estimate of Example 5.1:

$$\hat{D}_{\text{overall}} = \frac{\sum_{i=1}^n L_i \hat{D}_i}{\sum_{i=1}^n L_i}$$

Variance calculations proceed in the same manner as for line intercepts (Example 5.1)

Table 5.4. Line transect data from a lizard study. Flushing (r) and right-angle distances (x) for individual transects.

#1		#2		#3		#4		#5		#6	
r	x	r	x	r	x	r	x	r	x	r	x
46	46	91	51	137	29	60	43	51	23	91	16
82	26	42	25	21	17	67	34	42	14	74	58
59	10	36	32	84	25	51	18	109	57	57	10
42	36	126	88	62	25	68	37	120	43	101	40
40	35	43	15	79	37	55	13	60	32	74	72
		100	96	80	0	55	33	39	32	46	13
				70	0	168	90	81	32	46	15
				95	35	90	73	67	18	99	37
				95	32	78	25	55	0	87	0
				61	41			165	75	58	11
				58	42			269	33		
				24	13			269	25		
								85	35		
								168	98		
								50	0		
								83	4		
								42	27		
								75	10		
								104	0		
#7		#8									
r	x	r	x								
153	48	85	79								
112	45	112	55								
126	34	94	0								
61	45	78	15								
53	43	158	68								
78	0	153	72								
53	17	153	74								
59	49	42	27								
78	64	42	27								
150	146										
128	34										
114	38										
90	54										
93	24										

To illustrate the use of right-angle distances, we use the half-normal distribution. This requires the assumption that right-angle distances from the transect line have the relative frequency given by:

$$f(x) = \frac{2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

This is just the familiar normal distribution, but with $\mu = 0$, i.e., centered on the transect line. Also, the distribution is multiplied by a factor of 2 in order to permit "folding-over" half of the distribution, and thus considering observed distances as though they all fell on one side of the transect line. Recalling that the general form

of the density estimator for line transects calls for an estimate of $f(0)$, we get:

$$f(0) = \frac{2}{\sqrt{2\pi} \sigma}$$

The parameter (σ) is estimated just as it is for the normal distribution, i.e.,:

$$\sigma_i = \left(\sum_{j=1}^m \frac{x_j^2}{m} \right)^{1/2}$$

With the exception that the divisor is m , rather than $m-1$, since in this case the mean is known (i.e., is zero). Inserting the above expression for $f(0)$ in Eq. (5.7) gives:

$$\hat{D}_i = \frac{m}{L_i \sqrt{2\pi} \hat{\sigma}_i}$$

The individual transect estimates appear in Table 5.4, and are combined just as with Hayne's method above:

$$\hat{D}_{\text{overall}} = \frac{\sum_{i=1}^n L_i \hat{D}_i}{\sum_{i=1}^n L_i} = \frac{0.822}{500} = 0.0016.$$

Table 5.5. Summary of line transect data for a lizard study.

Transect number	Length L_i	Number of observations (Hayne)	$L_i \hat{D}_i$	$L_i \hat{D}_i$ (half-normal)
1	30	5	0.050	0.061
2	50	6	0.052	0.040
3	60	12	0.109	0.169
4	80	9	0.066	0.076
5	100	19	0.129	0.197
6	80	10	0.074	0.113
7	60	14	0.083	0.100
8	40	9	0.056	0.066
	<hr/> 500	<hr/> 84	<hr/> 0.619	<hr/> 0.822
Density estimates			0.00124	0.00165

5.8 A "distribution-free" method

The terms, "parametric models" and "non-parametric models" have been used in the literature to classify line transect methods. We prefer to avoid that classification because the procedures thus far used mostly do involve parameter estimation. Hence we prefer to label the method presented here as "distribution-free," since it does not require the specification of a particular frequency distribution or "visibility curve." Burnham and Anderson (1976) suggest some other approaches that do not depend on a specific frequency distribution.

The method presented here is one originally devised by Cox (1962, 1969) and adopted for right-angle line transects by Eberhardt (1978b, 1979). A physical analogy, "length-biased sampling" was described in Section 5.4, in

reference to line-intercept methods. In that situation the physical size (length) of an object determines the probability that it will be intercepted by a randomly located transect line. However, a much larger class of situations may be included if one considers what Patil and Rao (1978) have described as "weighted distributions." They derive an equation of the form of Eq. (5.5) by supposing that the true frequency distribution cannot be observed directly, and that the observed frequency distribution is somehow "weighted" in the observation process.

In line transect work, the weighting function is what we have called a visibility curve above (cf. Fig. 5.8). Given random location of transect lines, the probability that an object will actually exist at a right-angle distance, x , from the transect lines is given by a uniform distribution. That is, theoretically, any distance is equally likely. However, the distances we actually observe depend on the visibility curve. Hence objects directly on the transect line are seen with certainty ($g(0) = 1.0$), while those at a considerable distances are seen very infrequently. Hence, formally, Eq. (5.5) should be written as:

$$f(x) = \frac{g(x)dx}{\mu} \quad (5.8)$$

so that dx represents the uniform probability that an object exists at any distance x from the transect line, and $g(x)$ is the "weighting function."

The main value of all of this is theoretical, in that it lets us extend the rather concrete notion of a line intercepting an object to the more abstract notion of a visibility curve. Further details and applications to a wide range of problems can be found in Patil and Rao (1978) and in the references cited in that paper.

Cox's method depends on tallying observed distances within fixed intervals away from the transect line. Thus all of the observations within a distance, Δ , on either side of the line are added up and used to estimate the true proportion of all observations, denoted $p(0, \Delta)$, that fall in that interval. Hence if there are k_1 observations within the distance Δ , we estimate $p(0, \Delta) = k_1/m$. Similar estimates are constructed for $p(\Delta, b\Delta)$, the next pair of parallel belts (Fig. 5.9) and $p(b\Delta, d\Delta)$. Cox's original method used only two intervals, but an extension to three or more intervals is readily obtained (Eberhardt 1979). However, it appears that the variance of the resulting density estimate increases as the number of intervals is increased (Eberhardt 1979), so we will limit the present discussion to two intervals.

An estimator for two intervals is (Eberhardt 1979):

$$\hat{\left(\frac{1}{\mu}\right)} = \frac{(b^2 - 1)\hat{p}(0, \Delta) - \hat{p}(\Delta, b\Delta)}{b(b-1)\Delta} \quad (5.9)$$

where Δ is the width of the inner interval and $b\Delta$ is the width of the inner two intervals (Fig. 5.9). The quantities $\hat{p}(0, \Delta)$ and $\hat{p}(\Delta, b\Delta)$ are estimated as described above, i.e.,

$$\hat{p}(0, \Delta) = \frac{k_1}{m} \quad \text{and} \quad \hat{p}(\Delta, b\Delta) = \frac{k_2}{m} \quad (5.10)$$

where there are k_1 objects observed within the belts of the width Δ on either side of the transect line, k_2 are seen within the two intervals (right- and left-hand sides of the transect line, Fig. 5.9) demarcated by Δ and $b\Delta$, and m is the total number of objects observed regardless of distance from the transect line.

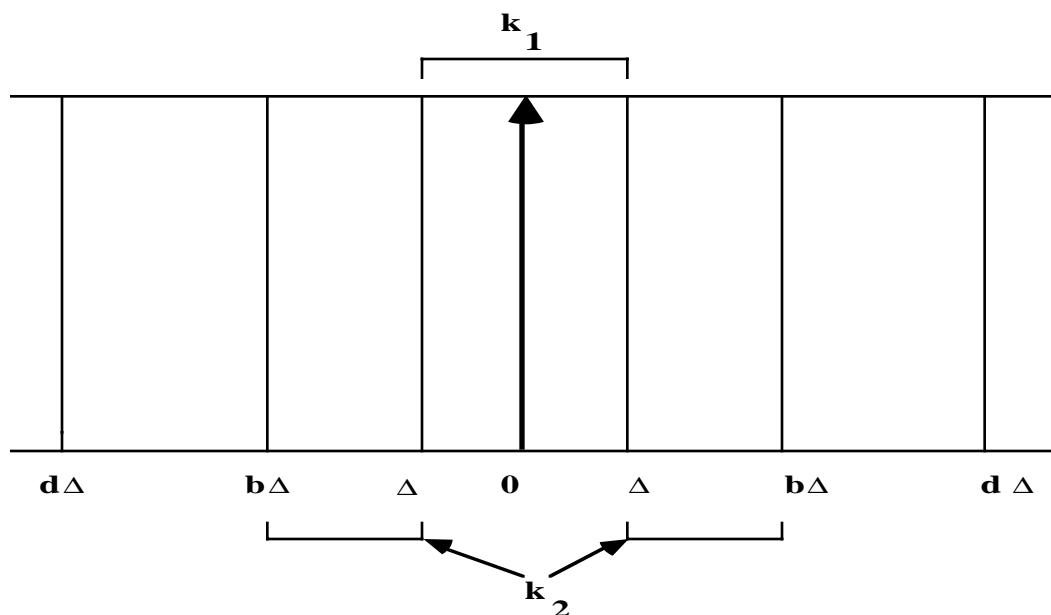


Fig. 5.9. Intervals or "belts" used in Cox method.

Having estimated the reciprocal of μ , all one needs to do is insert that estimate in Eq. (5.6) to estimate density per unit area, where the units are those in which right-angle distances (x_i) and length of transect (L) are recorded. Cox (1969) used $b = 2$, so that the inner and outer intervals are equal (i.e., they are both of width Δ). Since m appears in the numerator (Eq. 5.6) and in the denominators of $p(0, \Delta)$ and $p(\Delta, b\Delta)$, it effectively cancels out in the calculations. Hence, if one combines Eqs. (5.6), (5.9) and (5.10), the result is:

$$\hat{D} = \frac{(b+1)k_1 - k_2}{2Lb\Delta} \quad (5.11)$$

where we have used the result that $(b^2 - 1) = (b + 1)(b - 1)$. One apparent consequence of this simplification is that m (the total number of objects) is not required for density estimation. However, that quantity is essential in studying variability of the estimates, and thus should be recorded, except in special circumstances. One such situation may be in cases where identification of objects beyond a distance of $b\Delta$ is uncertain. Calculations are discussed in Example 5.7.

An interesting variant of Cox's method is the case where only one interval, of width Δ (on each side of the transect line), is used. This reduces Eq. (5.11) to:

$$\hat{D} = \frac{k_1}{2L\Delta} \quad (5.12)$$

We then have simply a strip transect, or "Kelker's method," in which it is assumed that all of the objects are observed within a strip of width 2Δ . It may

be noted that if the number of objects observed in the two belts is equal, i.e., if $k_1 = k_2 = k$, then Eq. (5.11) becomes:

$$\hat{D} = \frac{(b+1)k - k}{2Lb\Delta} = \frac{k}{2L\Delta}$$

so that we again have a strip transect.

Another variant, of interest to ornithologists, is Emlen's (1971) method. It turns out (Eberhardt 1978b:15) that Emlen's method essentially reduces to use of Kelker's method, or a strip transect. Details appear in Example 5.8. Since the assumption that all animals (or objects) are seen in the inner strip is an important and uncertain item, we recommend that two intervals be used in practice.

Variance estimation for the Cox method is a subject that needs more research, particularly research supported by field data. The weak point in present theoretical approaches is that they assume that the number of individuals observed (m) is Poisson-distributed, which essentially amounts to assuming random distribution of individuals. Since this is not likely to occur in practice, the present variance estimates are likely to be too low, i.e., underestimates. For the Cox method with two intervals, a convenient expression of the variance is obtained as (Eberhardt 1979):

$$[C.V.(\hat{D})]^2 \doteq \frac{(b+1)^2 k_1 + k_2}{[(b+1)k_1 - k_2]^2} \quad (5.13)$$

where b , k_1 , and k_2 are as defined above, and $C.V.(\hat{D})$ stands for the coefficient of variation of the density estimate. For practical purposes, if $C.V.(\hat{D})$ equals, say 0.25, one can propose approximate confidence limits on an estimate as being the estimate ± 50 percent (i.e., we round the usual 95 percent normal curve "Z-value" of 1.96 to 2.0).

An alternative procedure for variance estimation is to employ the "replicate sampling" idea, i.e., to break the total sample down into random subsets, calculate a density estimate from each such subset, and obtain the variance estimate from the resulting set of independent density estimates.

For planning purposes, a rough approximation (Eberhardt 1978b) is:

$$C.V.(\hat{D}) \doteq \left(\frac{4}{m}\right)^{1/2} \quad (5.14)$$

Two examples on actual data (Eberhardt 1979) suggest this equation underestimates the results of Eq. (5.13) by roughly 10 percent. Seber (1973) and Eberhardt (1978b), using different approaches, suggest a comparable result for flushing-distances (sighting-distances, radial distances) to be:

$$C.V.(\hat{D}) \doteq \left(\frac{2}{m}\right)^{1/2} \quad (5.15)$$

We thus have an indication that the variance using right-angle distances and the Cox method may be much as twice that obtained for flushing distances.

Example 5.7 The "Cox" method

The right-angle distances of Table 5.4 can be used to illustrate the Cox method, as given by Eq. (5.11). The main problem lies in selection of the two intervals Δ , and $b\Delta$. My recommendation (cf. Eberhardt 1979) is to include about 80 to 90 percent of the observations inside $b\Delta$. If we let $b = 2$ and $\Delta = 35$, then $b\Delta = 70$. Although density can be calculated directly from Eq. (5.11), in this case it would be desirable to make the intermediate calculation represented by Eq. (5.9), for use in comparison with the results of Example 5.6 above. This is because the transect lengths used in the example are artificial, so the best comparison is to estimate $(1/\mu)$ or $f(0)$. Hence the entries under sums of $L_i D_i$ from Table 5.5, are best compared with the estimate obtained from the Cox method. Note, however, that these quantities need to be doubled for comparison. Calculations with the Cox method can be carried out transect by transect, and it is probably worth doing so on Table 5.4 just to see how the estimates behave. With small samples, however, it is preferable to make a single estimate for the entire area (i.e., combine all of the observations in Table 5.4).

Example 5.8 Emlen's method

A method due to Emlen (1971) became quite popular with ornithologists. It depends on a "coefficient of detectability" which is determined by an intensive study on one area, and then used to adjust counts in other areas. The basic approach is to use the observed data to determine where visibility drops off. If we let this point be Δ , the assumption is that all birds are seen between the transect line and Δ . Suppose k_1 birds are counted in this strip, and that we want to estimate the number of birds expected to be found between the transect line and some outer boundary, R . The logical estimate is just $(k_1/\Delta)R$. Emlen divides the total number of birds (k) seen between the transect line and R by this projected number and calls this the "coefficient of detectability":

$$C.D. = \frac{k\Delta}{k_1 R}$$

This clearly amounts to an estimate of the proportion seen of the birds present between R and the transect line. Emlen then proceeded to divide the number seen (k^*) on a new area by the coefficient of detectability, and regarded this as a population estimate for the new area:

$$\hat{N} = \frac{k^* k_1 R}{k \Delta}$$

Actually R is selected so that a fixed transect length (1 mile) gives \hat{N} as the number of birds per 100 acres. Hence \hat{N} is really a density, and we note that if units of feet are used $2RL = 100(43,560) = C$, so we can write $R = C/2L$ and express \hat{N} as:

$$\hat{N} = \frac{Ck^*}{k} \left[\frac{k_1}{2L\Delta} \right]$$

Since the quantity in brackets is just Kelker's estimate [Eq. (5.12)], Emlen's procedure turns out to have the following steps:

(1) Estimate density on one area by Kelker's method, while counting all of the birds visible on 100 acres (k).

(2) Count all of the birds visible on 100 acres (k^*) on a new area.

(3) Use the ratio k^*/k to project the Kelker estimate of the first area to the second area.

Several limitations of the method seem apparent. One is that the visibility curve is assumed to have a particular form, i.e., all birds are seen out to some particular distance. A second limitation is that it is assumed that we can locate that distance from observed data. A third, and major problem, is that it is assumed that the visibility curve remains constant from area to area, and time to time.

These several limitations can be avoided simply by taking one precaution. That is to record separately all birds seen between the transect line and the distance Δ on the second area. One then can use Eq. (5.11) with $b = R/\Delta$ and get a direct estimate of density independently on the two areas.

5.9 Assumptions underlying line transect methods

A decision to use a particular line transect method needs to take into account the underlying assumptions. The list given here is based on seven assumptions given by Gates et al. (1968) and Seber (1973,1982), but is rearranged to show just which assumptions are required for a given line transect method. We assume that randomly located transect lines are utilized, and thus drop one restrictive assumption, that of random location of the objects being censused, which is not required for randomly located transect lines (Eberhardt 1978b).

The first three assumptions are basic and whether or not they are met will depend on behavior of the observer and of the animal being censused. They are:

- (1) No animal (or object) is counted more than once on a given transect line.
- (2) When flushed, each animal is seen at the exact position it occupied when startled by the observer's approach. Obviously this does not apply to animals or objects that are fixed in place during the census.
- (3) The response behavior of the population on a census plot does not change during the course of running a given transect.

Definitions of the response behavior serve to distinguish the various methods. One of these is achieved by defining a visibility curve as follows: The probability that an animal, or object, being seen, given that it is at a right-angle distance x from the transect line is a simple function, $g(x)$, such that $g(0) = 1$ (i.e., animals, or objects directly on the transect line are observed with certainty). These assumptions then suffice for right-angle line transects. Assumption (3) now means that the visibility curve does not change during the course of running a given transect line.

A fourth assumption serves to define conditions for the fixed flushing-distance line transect. This assumption defines the response behavior of the animals:

(4) Individual animals have fixed flushing radii, during the course of running a given transect, and flush if, and only if, an observer comes within this characteristic distance (r). It is of course, also necessary to assume that r is accurately observed and recorded.

An alternative assumption, plus some others, leads to the variable-distance line transect:

(4a) The animals are homogenous with regard to their inherent response behavior.

(5) The sighting of one animal is independent of the sighting of another.

(6) The instantaneous probability of flushing is a function, $f(r)$, of the radial distance, r , between animal and observer.

These several assumptions lead to the theory of the variable distance model (Eberhardt 1978b). It may be remarked that one could assume a particular mathematical model for $f(r)$ and proceed to derive efficient estimates for density estimation under such a model. This has been done in some of the literature (e.g. Gates et al. (1968) and Gates (1969)). We have previously mentioned two tests that should be applied to observed data before the flushing-distance method and Eq. (5.4) is used (cf. Example 5.5). We will return to discussion of some aspects of the above assumptions in a subsequent section on sampling design.

5.10 Strip transects

The simplest case of a strip transect occurs when the objects being censused are readily visible and sufficiently abundant to permit using a restriction on width of the strip covered. The method then amounts simply to a sample survey using long, narrow plots. The methods of Chapter 4 can then be applied. A basic assumption is that all of the objects on the plot are tallied.

When there is a reason to believe that not all of the objects on the plot are seen, then it may be necessary to introduce a visibility-curve. We have preferred to treat such situations under the heading of right-angle line transects (as in Fig. 5.1). However, this is mostly a matter of preference, and one could classify such situations as "strip transects using visibility-curves." This may be a more natural-seeming description in circumstances where a finite boundary exists on strip width. An example is the study of Anderson and Pospahala (1970), who counted duck nests on dikes. The width of the dike then provided a natural boundary on strip width. However, the methods of estimation will remain essentially those treated here as right-angle line transect, except that the total number of objects tallied (m) will be those tallied within the strip, and the visibility-curve, $g(x)$, is truncated (cut-off) at the strip boundary.

An important issue in such situations is that a visibility-curve has to enter the calculations of density in some manner. Several publications have used a procedure to correct for reduced visibility away from the transect line. This consists of summing the total observations from the entire survey for various intervals out from the transect line. With the symbols used above, one would thus have k_1 observations in the interval $(0, \Delta)$, k_2 in $(\Delta, b\Delta)$ and so on. It is then assumed that all of the animals are seen in the innermost interval, and the fraction seen in the other intervals is calculated from the observed data, i.e., k_2/k_1 , k_3/k_1 , etc. These rates are then used to adjust daily (or weekly, etc.) observations in the outer intervals, supposedly correcting them for the fraction missed. However, this procedure simply results in adjusting all of the intervals to equal the central one (to k_1 observations), so one may as well use only that interval and not bother with the rest. The same kind of procedure has also been used to adjust for numbers seen by time of day when certain observation periods give the highest counts.

While the Cox method (Sec. 5.8) does not require postulation of a specific visibility-curve, it does take the existence of such a curve into account in the estimation procedure. All of the other methods actually used thus far do specify a particular curve or "family" of curves.

Strip transects have been widely used in aerial surveys, largely of terrestrial animals. It is now well-established that not all of the animals on the strip are seen by the aerial observers. Caughley (1974) has summarized evidence on this point. Caughley et al. (1976) have conducted some experiments designed to explore the effects of strip width, altitude and speed on the numbers of animals counted. They go further, and use multiple regression equations to attempt to correct for these variables. However, I do not recommend the use of such equations, because a very uncertain sort of extrapolation is utilized--going from the observed data to zero strip width, speed, and altitude.

Two alternatives seem worth consideration. One is to utilize such experiments to arrive at a standard set of observation parameters, and to then regard the observed data as an index. When accurate counts by another method are feasible, one can then attempt to go further by "ground-truth" correction. A variety of special precautions need to be taken in aerial surveys, and are described in a publication by Norton-Griffiths (1975).

As mentioned in Sec. 5.8, the Cox method might be applied to aerial surveys in the form of Eq. (5.11), using two strips. This approach is particularly attractive in that it will not ordinarily be possible to attempt to record right-angle distances. About all that is likely to be feasible is to record observations in two intervals, demarcated by markers on windows and struts. Since most such surveys are conducted by observers who look out side windows of small aircraft, a particular drawback to this arrangement needs to be noted. This is that the visibility-curve is not likely to be that of Fig. 5.8, with certainty of observation of animals directly on the transect line. This is because the transect line is directly under the aircraft, and not readily viewed by the observer.

Unless a specially fitted-out aircraft is available, with provision for a "bow" observer to look directly forward and down, the only alternative seems

to be to attempt to locate, by experience, the right-angle distance that can be viewed effectively and comfortably by observers. The observers should then concentrate on "guarding" (covering) a fairly narrow strip starting at that point. This strip then becomes the interval $(0, \Delta)$ and frequent glances up and out are used to tally animals in the outer strip $(\Delta, b\Delta)$. The critical point is to concentrate enough effort on one line which is considered to be "the" transect line (normally there will be one such line on either side of the aircraft). In larger aircraft, it may be feasible to assign two observers to a side. One does nothing but scan the "track line," while the second observes the outer strip $(\Delta, b\Delta)$.

5.11 Modified strip transects

Three modifications of strip transects have been mentioned above. One includes corrections for decreasing visibility with distance from the transect line, and we have elected to cover this situation under right-angle line transects. A second is the case where animals, largely marine mammals, are visible only intermittently. The third has to do with animals that are in fairly constant motion, as with some small birds.

One basis for dealing with animals that submerge, and thus are not always visible on a transect, assumes a constant diving time (u) and a constant period on the surface (s) between dives. This is not particularly satisfactory, since both quantities may vary, and needs modification. In shipboard counts of ringed seals (*Phoca hispida*), McLaren (1961) assumed that all surfaced seals could be seen out to a fixed distance (r) from the vessel. This, too, is not a very reasonable assumption, as quite certainly there will be a decrease in visibility with distance. It might, however, be acceptable if this distance (r) is kept reasonably short. If the average probability (\bar{p}) that a seal within a strip of width $2r$ will be counted can be estimated, then the observed count (m) within the strip can be translated to an estimate of density as:

$$\hat{D} = \frac{m}{2rL(\bar{p})} \quad (5.16)$$

i.e., the number present is estimated as m/\bar{p} and this is divided by the area of a strip of length L .

McLaren (1961) gave the probability (\bar{p}) of seeing an individual seal, given that it is in the strip, as:

$$p = \frac{t}{s+u} + \frac{s}{s+u} \quad (5.17)$$

where t denotes the duration of the period when a surfaced seal would be visible to an observer. This varies according to the right-angle distance from the vessel, since the radius of visibility (r), shown in Fig. 5.10 limits the time an animal can be seen at a given right-angle distance. If v denotes velocity of the vessel, then (see Fig. 5.10):

$$t = \frac{y}{v} = \frac{r \sin \theta}{v}$$

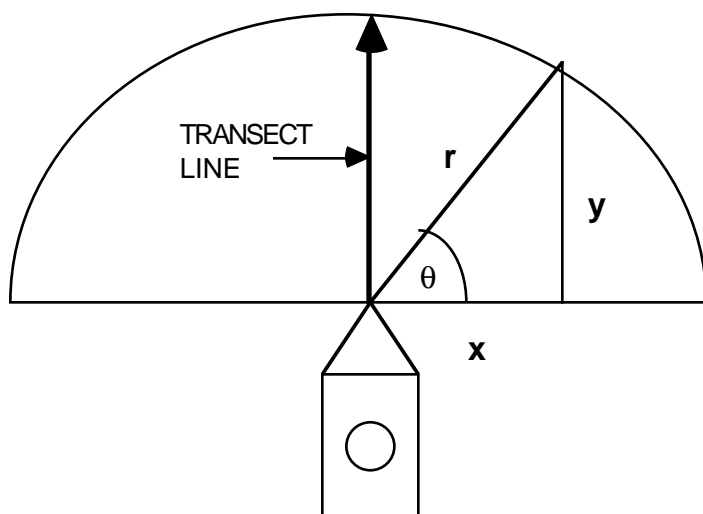


Fig. 5.10. Dimensions used in shipboard surveys of marine mammals. The arrow denotes the ship's course along a transect line, while the observers scan the semi-circular area of radius r .

McLaren assumed that no point could be kept under observation for as long as u minutes, that is, $t < u$. Given this restriction, Eq. (5.17) can be obtained by considering the two successive intervals representing a dive (u) and the succeeding time on the surface (cf. Exercise 5.13.1). If we make the usual assumption of random transect locations (and thus a uniform probability of a seal being present at a given right-angle distance), then Equation 5.17 can be "averaged" to obtain:

$$\bar{p} = \frac{\pi}{4} \frac{r}{v(s+u)} + \frac{s}{s+u} \quad (5.18)$$

This differs from McLaren's (1961) result (see Eberhardt 1978b).

A similar approach has been employed for censusing whales visually, except that a narrower width of field forward of the vessel is scanned. Also, much longer detection distances are postulated, due to the greater visibility of "spouting" or "blowing" by the whales. Doi (1974) developed an expression for the probability of detection. He also assumed a constant diving time, so that the same question of the effect of a variable diving time arises in connection with his results. An important difference in the two approaches is that McLaren assumes $t < u$, i.e., that submerged seals may go undetected, even if they are in the zone of maximum possible detection directly ahead of the vessel. Doi, however, postulated a zone within which the observation time is long enough that any whales were seen with certainty. Doi also introduced a correction factor (K) for the prospect that observers may fail to see some whales, even though they do surface and "blow." This factor is:

$$K = 1 - \left(1 - \frac{\theta_p}{2\theta_1}\right)^s$$

where θ_p represents the visual angle of the observer, θ_1 is the angle searched (on either side of the vessel) and s is the number of observers. Buckland et al. (1993) described another method for whales called "cue counting".

The second modification to be considered here is that required to deal with animals, such as small birds, that are in motion during the course of the survey. Yapp (1955) proposed an approach based on the kinetic theory of gases. The mathematical aspects were later reviewed by Skellam (1958). The two equations involved are:

$$\hat{D} = \frac{z}{2rv} \quad (5.19)$$

where D = density of the population, z = number of encounters per unit time, v = average velocity of the animals relative to the moving observer, and r = range or radial distance within which an animal must approach the observer to effect an encounter.

$$v^2 = \bar{u}^2 + \bar{w}^2 \quad (5.20)$$

where \bar{u} = average velocity of the organisms and \bar{w} = average velocity of the observer.

An important assumption is that the behavior of the animals is not influenced by the presence of the observer. Eq. (5.19) is based on the assumption that the area in which encounters take place is a circle of radius r . This, then, is the same sort of troublesome assumption encountered before in this chapter. If we let $z = m/T$, where m is the number of animals observed

during the total time of the survey (T), and also assume $\bar{u} = 0$, then:

$$\hat{D} = \frac{m}{2r\bar{w}T} = \frac{m}{2rL}$$

where L represents the total distance traveled by the observer. We thus have the usual equation for a simple strip transect. One can, of course, let

$\bar{w} = 0$, i.e., assume that the observer sits still and base results on the average velocity of the organisms:

$$\hat{D} = \frac{m}{2r\bar{u}T} \quad (5.21)$$

This has some attractive features, in that the radius (r) can probably be determined with reasonable accuracy under such circumstances, and a motionless observer is less likely to influence behavior of the animals. A drawback is in the "representativeness" of the spot selected for observation. No doubt random selection of several spots would help on this score. However, if the radii vary, as they likely will, then questions of the effect of density vs. cover type may need to be considered.

An important problem with the above method is that of measuring the average velocity of the animals (\bar{u}). Clearly this cannot be done during the survey, at least not if the observer is also moving. However, if the observer sits still he might then use a stop watch to time movements of animals and thus estimate their velocity.

Little use seems to have been made of Yapp's method, so that it is difficult to provide an evaluation based on experience.

5.12 Survey design

Although much of the discussion of line transect methods is couched in terms of results obtained on a single transect line, practical use of the method will frequently require combining results from a number of separate lines into a single sampling unit. This will be especially true in terms of variance calculations, since precautions need to be taken to avoid individual sampling units on which no animals are observed. As we have already indicated, we believe variance estimates based on theory should mainly be used for such purposes as comparing methods of estimation, appraising bias, and the like. Another important use is in obtaining approximations suitable for estimating sample sizes in planning a survey, as in Eqs. (5.14) and (5.15).

In the actual analysis of survey results, we recommend variances be estimated directly from the survey estimates, as illustrated in the several examples. However, the investigator should not wait until the data are all in hand before considering how this is to be done. The arrangements for analysis of the data should instead be decided at the survey design stage.

Usually the survey will require a number of days for completion so that a worthwhile precaution is to arrange the sampling plan so that the transects run in the same sub-area are spread out over the total time interval during which the survey is conducted. Thus "replication in time" is introduced into the survey, and it may be useful, in analysis of the data, to try to evaluate any trends in time. When this kind of arrangement is feasible, it will be important to randomize the locations of successive lines falling in the same sub-area. In fact, this may well be the best way to use randomized sampling, in that the separation in time will usually eliminate the need for concern about having two transects fall close together. When large areas must be dealt with, it will usually not be possible to use a scheme of this sort.

The above scheme may be illustrated by reference to Fig. 5.11, which shows a study area divided into three subunits, denoted by vertical lines in the figure. One randomly located transect line (L_1 , L_2 , and L_3) is shown in each sub-area for each day on four successive days. To obtain a variance estimate on the basis of "interpenetrating," or "replicate" sampling, one simply calculates an estimate of density for each day, and uses that estimate in the variance calculation. That is, density is estimated as

$$\hat{D} = \frac{1}{n} \sum_{i=1}^n D_i \quad (5.22)$$

and variance as

$$s^2(\hat{D}) = \frac{1}{n(n-1)} \sum_{i=1}^n (D_i - \hat{D})^2 \quad (5.23)$$

where in this case $n = 4$. Note that the variance given is that of a mean, i.e., $s^2(\hat{D})$ is usually described as the standard error.

For larger areas, travel time will be rather too costly to permit use of single transects in each sub-area. Alternatives are to use several randomly located lines in each sub-area, or to use a systematic arrangement with a randomly selected starting point. Suppose three lines in each unit are to be used. Then each base line length (W_i) is divided into three segments of length $W_i/3$. A random location is selected in the first and the remaining units spaced out by the interval $W_i/3$. It may be noted that the baselines (W_i) of Fig. 5.11 are not of the same length. This is because of the irregular shape of the region--it is best to try to keep the areas of the subunits about equal. Differing lengths of transect line can be handled as described in Example 5.1. Note that the three systematically placed lines discussed above should be treated as one transect line in the analysis.

Stratified random sampling (Chapter 4) may well be desirable in line transect work. Example 4.6 illustrates use of stratification with a strip transect. Unfortunately stratified sampling has not been used much with line transects, so we have little experience to draw on for planning. One prospect is that the use of variable sampling intensity (by strata) will call for making individual population estimates for each stratum. Obtaining separate variance estimates for each stratum may thus require fairly intensive sampling in each stratum.

A very important feature of survey design is to review the underlying assumptions (Sec. 5.9), and to consider whether the proposed design is likely to result in violation of one or more of the assumptions. I have repeatedly recommended random sampling, as this is the basis for the present theory. A practical alternative is a systematic sample with a random start. With animals that are highly mobile, one has to avoid a sampling pattern that places lines near enough together that individual animals might be seen twice on the same systematically arranged sampling unit.

The various assumptions that have to do with response behavior obviously require good knowledge of the species and situation. Some species behave in ways that make them doubtful candidates for line transect censusing. When right-angle distances are used, the "behavior" of the observer is of crucial importance. Some design arrangements can help reduce the effect of observer differences. For example, if several observers are used in a single aircraft, for an aerial survey, they should rotate through the viewing positions fairly often (in small aircraft this may be practicable only on landing). This practice helps "average out" observer and position effects.

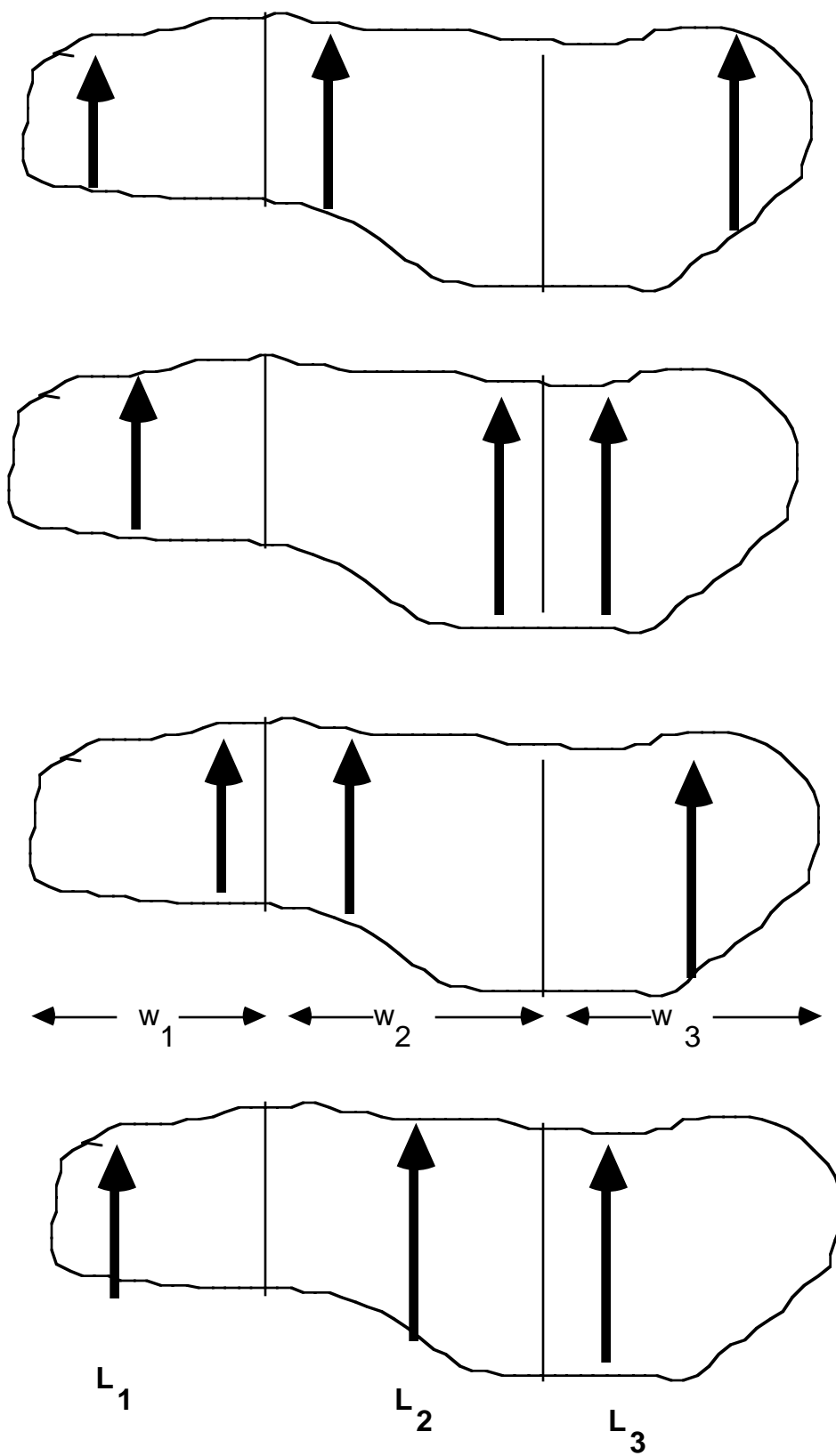


Fig. 5.11. Randomization of transect lines within sub-areas on successive census days.

Since circumstances of line transect surveys are quite variable, it is not possible to review each situation with regard to the assumptions. The investigator needs to understand them, and to take precautions wherever possible. Another example is that lines should not be run close to sharp breaks in cover, topography, etc. One can, of course, simply not census on the side of a line that parallels such a break. Sometimes it is possible to arrange that the lines go at right-angles to such "discontinuities," and this should help. A similar reasoning dictates that transect lines should not run the "long way" of an elongated study area. Methods to minimize errors of measurement and data-recording are of course essential in survey design and planning.

A variety of recommendations concerning transect methods in censusing marine mammals appears in Eberhardt et al. (1979). Some of these may be useful in other circumstances. As noted earlier, much more detail on recently developed methods appears in the book by Buckland et al. (1993).

5.13 Exercises

5.13.1 Calculate $CV^2(p)$ from Table 5.2 using ratio estimation, and calculate a standard error for p using the individual proportions. Show your calculations.

5.13.2 calculate the weighted average density from Table 5.2 using ratio estimation, and its standard error. Show your calculations.

5.13.3 Carry out calculations for the Hayne and half-normal methods for the lizard data of Table 5.4. Show calculations.

5.13.4 Do the calculations for Example 5.7.

5.13.5 Estimate the D_i for Exercise 5.13.3 and tabulate these along with the D_i from Exercise 13.4. Compute coefficients of variation treating each transect as an independent estimate. Compute correlations among the three sets of data. Also compute the coefficient of variation for the Cox method given in Eq. (5.13), combining the data from the several transects.

5.13.6 Components of variance

It was remarked in Sec. 5.5 that the fixed distance model could be used to show that a larger variance results if right-angle distances are used for estimation, rather than the flushing distance. One way to appreciate this is to recall that the basis for estimation for the fixed distance model depends on doubling the flushing-radius to determine the probability of observing that individual. Doing this with the right-angle distance introduces an additional component of variability due to the fact that the observed right-angle distance (x) falls randomly between zero and the flushing-distance (r). Students with some training in mathematical statistics may want to try calculating coefficients of variation for x and r , assuming x to be uniformly distributed on $(0,r)$ and that r has some underlying distribution, say $m(r)$. One can then find the two C.V.'s in terms of the first 3 moments (μ_1, μ_2, μ_3) and obtain a notion of the relative difference in efficiency.

5.13.7 Deriving a sighting probability

Students should attempt to derive Eq. (5.17), assuming s and u are fixed and that $t < u$. Nothing beyond elementary probability considerations is involved, but a diagram helps.

5.13.8 A seal census

McLaren gives data as follows. Ship's speed 0.12 nautical miles per hour, visibility limit 0.32 miles, $s = 1$ minute and $u = 3$ minutes, and 43 seals were counted on a given transect. He does not give the transect length. Students should convert Eq. (5.16) to represent number of seals (N) in the area scanned by the observer and carry out the relevant calculations.